A Bootstrap Granger Causality Test from Exchange Rates to Fundamentals

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Abstract—This study is in response to claims of a Granger causality relationship from exchange rates to fundamentals. The alleged relationship is based on an asymptotic test performed in a previous study, which has been taken as evidence to support the present-value model for exchange rates. We adopted a bootstrap method to reassess the evidence supporting Granger causality, the results of which contradict the findings of the previous work. A further Monte Carlo experiment suggests that the causality test implemented in the previous study tended to spuriously reject null hypotheses, implying that the existing evidence for the present-value model of exchange rates is very weak.

Keywords—bootstrap; Granger causality; the present-value model; exchange rates

I. INTRODUCTION

Over the past two decades, many researchers have sought empirical evidence regarding the relationship between exchange rates and the fundamentals implied by theoretical models. However, evidence of the relationship found in the literature has rarely been significant. The weak relationship between exchange rates and macroeconomic aggregates, such as monetary supply and output as well as the difficulty in linking exchange rates to other aspects of the economy are all part of the “exchange-rate disconnect puzzle [13].”

The existence of exchange rate predictability has been an issue since, in the seminal paper, references [11] and [12] claimed that a simple random walk model outperformed all of the structural models they tested in the out-of-sample forecast for exchange rates. Their findings imply that changes in exchange rates are nearly unpredictable, which has inspired many researchers to conduct further investigation into the predictability of exchange rates. Unfortunately, researchers have had a hard time providing a satisfactory explanation as to why it has been so difficult to beat random walk forecasts of exchange rates. Although favorable evidence supporting the predictability of exchange rates was uncovered in empirical studies by a number of researchers, such as [9] and [10], evidence denying such predictability has also appeared in studies by [3] and [6].

In a recent paper, reference [5] proposed a new approach to the present-value asset-pricing model, in which they explain near-random walk exchange rates. In the present-value model, exchange rates are determined by a linear combination of observable as well as unobservable fundamentals. Reference [5] showed that exchange rates resemble a random walk when at least one fundamental follows a unit root process, and when the discount factor of the present-value model approaches one. The existing empirical evidence regarding near-random walk exchange rates is only an implication of the present-value model—neither refuting it, nor supporting it. Direct validation of the present-value model is required. To provide such validation, reference [5] evaluated the Granger causality relationship between exchange rates and fundamentals implied by the present-value model.

Reference [5] implemented the asymptotic method to test the Granger causality relationship between exchange rates and fundamentals, using a vector autoregression (VAR) model. They uncovered a number of statistically significant Granger causality relationships from exchange rates to fundamentals, implying that exchange rates are useful for predicting fundamentals. Moreover, these findings are consistent with the implications of the present-value asset-pricing model for exchange rates and may fundamentally alter the debates concerning exchange rates.

Nevertheless, the asymptotic test inferences in [5] were constructed from three types of sample periods and the size of two of the sample periods was relatively small. It is well known that the asymptotic test method suffers from small-sample problems in many applications. These results motivated us to re-evaluate the evidence supporting the present-value model by implementing other testing methods.

This paper employs a bootstrap method to reassess the existing evidence of the Granger causality relationship between exchange rates and fundamentals. Thanks to advances in computer technology, the bootstrapping technique can be implemented quickly. The bootstrap method generally shows a lesser degree of size distortion and provides more precise test inferences than the asymptotic method, when the sample is small. In a prominent study by [9], the method based on conventional asymptotic theory was replaced with a bootstrap simulation method to deal with bias and problem of size distortion in testing the performance of macroeconomic models in forecasting changes in long-horizon exchange rates. Reference [9] drew test inferences based on bootstrap empirical distribution to present evidence supporting the possibility of predicting long-horizon exchange rates.

To compare the new test results from the bootstrap distribution with the existing test results from the asymptotic distribution, we used data identical to that used by [5]. The results obtained from the two test procedures became more distinct when the sample size was smaller. In particular, with smallest sample, fourteen of the thirty null hypotheses stating that exchange rates do not Granger cause fundamentals, were
rejected at the 10% significance level in the asymptotic tests, while only five out of thirty null hypotheses were rejected in the bootstrap tests.

A Monte Carlo experiment was implemented to examine the robustness of the two types of test methods and to investigate whether the bootstrap test outperformed the asymptotic test in this particular application. Results show that the size of the asymptotic Granger causality test was generally larger than that of the bootstrap test. The size of the bootstrap test remained at approximately 10% in all of the samples, but the size of the asymptotic test increases to nearly 40% when the sample size decreased. The increase in the size of the asymptotic test showed that the asymptotic test had been influenced by size distortion. This is a clear indication that the asymptotic Granger causality test tends to spuriously reject null hypotheses in this application.

The remainder of this paper proceeds as follows. In the next section, we present a review of the present-value asset-pricing model for exchange rates and briefly introduce the explanation of the present-value model for exchange rates in Section 6. In addition to the models for the exchange rate predict no better than a random walk model were proposed in [5]. In Section 4, the exchange rate under the present-value model takes the form:

\[ s_t = y f_t + \psi E_t s_{t+1} \]

\[ = y \sum_{j=1}^k \psi^j E_t f_{t+j} + \psi^{k+1} E_t s_{t+k+1} \] (1)

where

\( f_t = (m_t - m_t^*) - \phi (y_t - y_t^*) \)

\( y \equiv 1/(1+\lambda) \)

\( \psi \equiv \lambda y = \lambda / (1 + \lambda) \)

For the no-bubbles solution (\( \psi < 1 \)), the transversality condition, \( \lim_{k \to \infty} \psi^k E_t s_{t+k} = 0 \), holds. The latter term in the right hand side of equation (1) vanishes. The exchange rate, therefore, is the discounted present value of future monetary fundamentals. In contrast, for the rational bubbles solution, the transversality condition does not hold, and the exchange rate will eventually be governed by explosive bubbles. Yet, in the real world, even if the rational bubble occasionally dominates the exchange rate behavior, the bubble does not last for a long time. Therefore, the rational bubbles solution is not considered in the discussion throughout this paper.

II. EXCHANGE RATE UNDER THE PRESENT VALUE MODEL

A. The Conventional Monetary-Income Model for Exchange Rates

An exchange rate can be viewed as an asset price in the present-value model. The flexible exchange rate in the framework of the conventional present-value model is determined by the expectations of future observable fundamentals and the expectation of its own future value. To construct the present-value model for exchange rate, we denote the log of the nominal exchange rate measured at time \( t \) by \( s_t \) and denote the observable macroeconomic fundamentals of the nominal exchange rate measured at time \( t \) by \( f_t \). In the conventional money income model, the money market relationship is given by

\[ m_t = p_t + \phi y_t^* - \lambda i_t^* \]

\[ m_t^* = p_t^* + \phi y_t^* - \lambda i_t^* \]

The variable \( m_t \) is the log of the money supply, \( p_t \) is the log of price level, \( y_t^* \) is the log of income, and \( i_t \) is the level of interest rate. The asterisk represents variables in the foreign country. The parameter \( 0 < \phi < 1 \) is the income elasticity of money demand and \( \lambda > 0 \) is the interest rate semi-elasticity of money demand. The parameters of the foreign money demand are identical to the home country’s parameters.

Assuming the nominal exchange rate equals its purchasing power parity (PPP), we have:

\[ s_t = p_t - p_t^* \]

The financial market equilibrium is given by the uncovered interest parity (UIRP):

\[ i_t - i_t^* = E_t s_{t+1} - s_t \]

Here, \( E_t s_{t+1} \) is the rational expectation of the exchange rate at time \( t + 1 \). Putting the above equations together and rearranging, the present-value formula for the exchange rate takes the form:

\[ s_t = y f_t + \psi E_t s_{t+1} \]

\[ = y \sum_{j=1}^k \psi^j E_t f_{t+j} + \psi^{k+1} E_t s_{t+k+1} \] (1)

where

\( f_t = (m_t - m_t^*) - \phi (y_t - y_t^*) \)

\( y \equiv 1/(1+\lambda) \)

\( \psi \equiv \lambda y = \lambda / (1 + \lambda) \)

For the no-bubbles solution (\( \psi < 1 \)), the transversality condition, \( \lim_{k \to \infty} \psi^k E_t s_{t+k} = 0 \), holds. The latter term in the right hand side of equation (1) vanishes. The exchange rate, therefore, is the discounted present value of future monetary fundamentals. In contrast, for the rational bubbles solution, the transversality condition does not hold, and the exchange rate will eventually be governed by explosive bubbles. Yet, in the real world, even if the rational bubble occasionally dominates the exchange rate behavior, the bubble does not last for a long time. Therefore, the rational bubbles solution is not considered in the discussion throughout this paper.

B. The Explanation for the Present Value Model for Exchange Rates in [5]

Several explanations for why the conventional monetary models for the exchange rate predict no better than a random walk model were proposed in [5]. In addition to the observable fundamental in the conventional model, the exchange rate behavior is also influenced by the present value of the future unobservable fundamentals. According to [5], the exchange rate under the present-value model takes the form:

\[ s_t = (1 - b)(f_{1t} + z_{1t}) + b(f_{2t} + z_{2t}) + b E_t s_{t+1} \] (2)

where \( b \) is the discount factor, and \( z_{1t+j} \) is the unobservable fundamental at time \( t + j \). For the no-bubbles solution, after imposing the transversality condition, \( \lim_{k \to \infty} b^k E_t s_{t+k} = 0 \), on equation (2), equation (2) can be written as:

\[ s_t = (1 - b) \sum_{j=0}^{\infty} b^j E_t (f_{1t+j} + z_{1t+j}) \]

\[ + b \sum_{j=0}^{\infty} b^j E_t (f_{2t+j} + z_{2t+j}) \] (3)

Equation (3) provides two explanations for why it is so hard to beat the random walk model in predicting the exchange rate in the previous work. First of all, if the exchange rate is governed by the unobserved fundamentals, the exchange
rate change will naturally hard to be predicted. Secondly, as pointed out in [5], the value of discount factor value in equation (3) plays an important role in the exchange rate behavior. Under regular conditions, when the discount factor \( b \) is close to 1 and at least one fundamental has a unit root process, the correlation of the first-differenced exchange rate is close to zero. Therefore, the present-value model for exchange rates described by equation (3) implies that the exchange rate is close to a random walk.

According to [5], equation (2) is applicable to many macroeconomic models for exchange rates. We take the money income model as an example. Here, the exchange rate in the money income model equals its PPP value plus the real exchange rate (\( q_t \))

\[ s_t = p_t - p^*_t + q_t \]

In addition, consider a shock to the money demand (\( u_{mt} \)) in the money market and a deviation from rational expectations uncovered interest rate parity (\( \rho_t \)) in the international capital market. The money market relationship is:

\[ m_t = p_t + \phi y_t^* - \lambda t + u_{mt} \]

The interest parity relationship is:

\[ E_t s_{t+1} - s_t = i_t - i^*_t + \rho_t \]

The analogous foreign variables are \( m^*_t, p^*_t, y^*_t, i^*_t \), and \( u^*_{mt} \). The parameters of money demand are identical across countries. The exchange rate now can be expressed as:

\[ s_t = \frac{1}{1 + \lambda} \left[ m - m^*_t - \phi (y_t - y^*_t) + q_t \right. \]

\[ \left. -(v_{mt} - v^*_{mt}) - \lambda \rho_t \right] + \frac{\lambda}{1 + \lambda} E_t s_{t+1} \]

\[ (4) \]

Fitting equation (4) into the present-value model framework of equation (2), we can see that the discount factor of equation (4) of the money income model is \( \lambda / (1 + \lambda) \). The observed fundamental \( f_{kt} = m_t - m^*_t - \phi (y_t - y^*_t) \), and the unobservable fundamentals \( z_{kt} = q_t - (v_{mt} - v^*_{mt}) \) and \( z^*_{kt} = -\rho_t \). Under the explanation in [5], the money income model would imply the near-random walk exchange rate if the discount factor \( \lambda / (1 + \lambda) \) is close to one and at least one fundamental has a unit root process.

In the relevant literature, the value of the discount factor is suggested to be in a range between 0.97 and 0.98 for the quarterly data, which supports the claim of the near-one discount factor value for the near-random exchange rate in [5]. Hence, the present-value model with the close to one discount factor has an implication that the exchange rate approximately follows a random walk model. While the finding of the random walk exchange rate in the empirical studies corresponds to this implication of the present-value model for exchange rates, they can only be considered as evidence not against the model. There is still a need for direct evidence to validate the model.

### III. THE BOOTSTRAP GRANGER CAUSALITY TEST

According to [2], an asset price of the present-value model such as the stock price should help to predict its determinant fundamentals. Testing the Granger causality relationship between the asset price and its determinant variables, therefore, is helpful to find direct evidence for the present-value asset-pricing model. In [5], the authors conduct bivariate and multivariate Granger causality test to evaluate the present-value model for exchange rates. From the asymptotic tests, they find statistically significant Granger causality from exchange rates to fundamentals.

Nevertheless, the type of the data used in [5] makes the test results skeptical. They divide the full sample (1974:Q1 ~ 2003:Q3) into two subsamples in 1990:Q3 due to the evolution of European Monetary Union and the reunion of Germany’s economy during this period, and they use the full sample as well as the subsamples in the asymptotic Granger causality test. Yet, conducting the asymptotic test with those subsamples may lessen the credibility of the test result because the sample size of all subsamples is very small. For example, the data of France, Germany and Italy all are not available after 1999:Q1, and the number of observations in the later part of sample period (1990:Q3 ~ 2003:Q3) of those countries is merely 34. The small-sample problem is very likely to occur in this case, and we might need to find more evidence from other testing techniques such as the bootstrap method to confirm the existing evidence for the model.

In response to the asymptotic Granger causality test in [5], the goal of this article is to re-assess the existing evidence of the present-value model for exchange rates. Owing to advanced computer technology, the bootstrapping technique now can be quickly implemented. Moreover, the bootstrap method is generally believed having less size distortion and providing more precise test inferences than the asymptotic method in many applications if the available sample size is small. Thus, this study adopts the bootstrapping technique as an alternative test method.

#### A. The VAR Model for the Granger Causality Test

In order to compare the new test results from the bootstrap test with the existing result from the asymptotic test, the data used in this paper are identical to what were used in the asymptotic Granger causality test in [5]. In addition, this paper focuses on the bilateral relationship of the U.S. exchange rates against other six countries of G7 members. The fundamental measures include the money supply fundamental \( (m_t - m^*_t) \), the output fundamental \( (y_t - y^*_t) \), the PPP fundamental \( (p_t - p^*_t) \), and the UIRP fundamental \( (i_t - i^*_t) \). In addition, we also include the monetary fundamental. Following [9], the income elasticity, \( \phi \), in the money demand is set to 1, so the monetary fundamental \( (m_t - m^*_t) - (y_t - y^*_t) \). The exchange rate and each of the fundamental measures are shown to maintain root unit processes, so they are presented in the first-differenced form. The data do not show an explicit cointegration relationship between the exchange rate and each fundamental measure, so the VAR model does not include a vector correction term.
The bivariate VAR model in testing the Granger causality relationship between $\Delta s_t$ and $\Delta f_t$ takes the form:

$$\begin{align*}
\frac{\Delta s_t}{\Delta f_t} &= \frac{c_s}{c_f} + \sum_{i=1}^{4} \phi_{1i}^{11} (\Delta s_{t-i}) + \phi_{2i}^{12} (\Delta f_{t-i}) + \epsilon_{s,t}^* + \omega_{f,t}^* \\
\end{align*}$$

where $c_s$ and $c_f$ are the constant terms, and the innovation terms $\epsilon_{s,t}^*$ and $\omega_{f,t}^*$ are assumed to be independently and identically distributed ($i.i.d.$) with mean zero and have variances $\sigma_{s,t}^2$ and $\sigma_{f,t}^2$, respectively. The null hypothesis of the Granger causality test between the exchange rate and the fundamental restricts certain parameters $\phi_i$ in the equations in model (5) to be zero. For example, the null hypothesis of the non-Granger causality running from the exchange rate to the fundamentals restricts $\phi_{2}^{21}$, $i = 1, \ldots, 4$ to be zero.

**B. Bootstrap Test Algorithm**

Numerous bootstrap methods have been developed based on different properties of the time series process [7]; [8]; [1]. Because of the simple algorithm in generating bootstrap replications, the residual-based bootstrap has become the most popular one. Under the $i.i.d.$ error assumption, one can repeatedly draw residuals and generate the necessary pseudo data.

Since the Jarque-Bera test rejects the null hypothesis of the Gaussian innovation for most of the data used in this paper, all bootstrap test inferences are drawn from the test with nonparametric resampling method. The bootstrap algorithm for the Granger causality test consists of four steps.

Step 1. Given the original observation, estimate coefficients by the estimated generalized least square (EGLS) method for VAR model (5) under $H_0$ and $H_1$, respectively, and obtain the test statistic $\hat{\lambda}$.

$$\hat{\lambda} = T \left(\ln|S_{t_0}| - \ln|S_{t_1}|\right)$$

$S_{t_0}$ and $S_{t_1}$ are residual covariance matrices under $H_0$ and $H_1$, respectively.

Step 2. Apply the estimates $\hat{\phi}_i$’s estimated under the null hypothesis in step 1 to generate the pseudo-data $\{\Delta s_{t}^*, \Delta f_{t}^*\}$ with the same length as the original data $\{s_{t}, f_{t}\}$. For instance, if the null hypothesis is $\Delta s_{t}$ does not Granger causes $\Delta f_{t}$, the DGP is:

$$\begin{align*}
\Delta s_{t}^* &= \hat{\epsilon}_s + \sum_{i=1}^{4} \hat{\phi}_{1i}^{11} \Delta s_{t-i} + \sum_{i=1}^{4} \hat{\phi}_{2i}^{12} \Delta f_{t-i} + \epsilon_{s,t}^* \\
\Delta f_{t}^* &= \hat{\epsilon}_f + \sum_{i=1}^{4} \hat{\phi}_{1i}^{21} \Delta s_{t-i} + \epsilon_{f,t}^* \\
\end{align*}$$

To initialize this process, specify $(\Delta s_{t-0}, \Delta f_{t-0}) = (0, 0)$ for $j = 1, \ldots, 4$ and discard the first 500 created data. The pseudo innovation term $\epsilon_{*,t} = (\epsilon_{s,t}^*, \epsilon_{f,t}^*)^T$ is random and drawn with replacement from the set of observed residuals $\hat{\epsilon}_t = (\hat{\epsilon}_{s,t}, \hat{\epsilon}_{f,t})^T$ obtained from step 1. After repeating this step 2000 times, the 2000-bootstrapped samples are obtained.\(^2\)

Step 3. For each bootstrapped sample, re-estimate the coefficients in the VAR model (5), and construct the corresponding test statistic $\hat{\lambda}$ as in Step 1.

$$\hat{\lambda} = T \left(\ln|\hat{S}_{t_0}| - \ln|\hat{S}_{t_1}|\right)$$

Step 4. Use the 2000 test statistics $\hat{\lambda}$ obtained from the bootstrapped replications in step 3 to construct the empirical distribution and determine the $p$-value for the LR statistic $\hat{\lambda}$ of step 1.

**IV. EMPIRICAL RESULTS**

Bootstrapping test results are determined by the empirical $p$-value from 2000 bootstrapped samples, and the asymptotic test results are based on the $p$-value of the asymptotic $\chi^2$ distribution. Following [5], the full sample is divided into two sub-samples in 1990:3 due to major economic and noneconomic developments in this period. Tables 1 to 3 summarize $p$-values of the LR test statistics of the Granger causality test results from different test methods on each sample period.\(^3\)

**Full Sample (1974:Q1~2001:Q3).** Tables 1(a) and 1(b) illustrate the $p$-values of the LR test statistic in Granger causality test from the full sample data based on the standard asymptotic distribution and the empirical bootstrap distribution, respectively. Table 1(a) shows that, at the 10% significance level, ten out of thirty null hypotheses that $\Delta s_t$ fails to Granger cause $\Delta f_t$ based on the asymptotic distribution are rejected. There are no rejections for Canada and the United Kingdom, and no null hypotheses are rejected in the cases of the output fundamental $\Delta (y_t - y_{f_t})$. At the 1% significance level, we reject the null hypothesis in

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\(^{1}\) Several standard test statistics, such as the $F$ test statistic, the Wald test statistic, and the likelihood ratio (LR) test statistic, can be used in the bivariate Granger causality test. Since the standard $F$, Wald, and LR test statistics are the same asymptotically, this paper provides the test result from the LR test statistic.

\(^{2}\) This study refers to [4], choosing to bootstrap 2000 replications because the bootstrapping $p$-value of the test statistic $\hat{\lambda}$ constructed from 2000 replications differed only marginally from those constructed from 2500 or 5000 replications.

\(^{3}\) The asymptotic test in this paper is replication results of Engel and West’s Granger causality test.
three out of six countries when we investigate the Granger causality relationship running from the exchange rate to the PPP fundamental. The results based on the bootstrapping distribution are reported in Table 1(b). Table 1(b) shows seven rejections in thirty tests at the 10% significance level. The evidence of the causality running from the exchange rate to the fundamental is slightly weaker than what we have seen in Table 1(a).

**Early Part of the Sample (1997:Q1–1990:Q2)**. Table 2 summarizes the test results for the early part of the full sample. As shown in Table 2(a), the asymptotic test result shows more evidence that the exchange rate Granger causes the fundamental than the full sample. In Table 2(a), the null hypothesis that $\Delta f_t$ fails to Granger cause $\Delta f_{t-1}$ is rejected in ten cases at the 5% significance level and in three more cases at the 10% significance level. For the output fundamental $\Delta(y_t - y^*_t)$, we do not see any result showing that $\Delta f_t$ causes $\Delta f_{t-1}$ in the full sample, but one rejection is found in the early part of sample period for Japan.

However, the test results in Table 2(b) are very different from those in Table 2(a). The evidence of the Granger causality relationship running from exchange rates to fundamentals is weaker in the bootstrap test than in the asymptotic test. At the 5% significance level, only three cases of the non-Granger causality null hypotheses are rejected, and no rejections are found in the cases of the output fundamental $\Delta(y_t - y^*_t)$.

**Later Part of the Sample (1990:Q3–2001:Q3)**. Table 3 reports the test results for the later part of the full sample. In Table 3(a), the evidence that the exchange rate Granger causes the fundamental is statistically significant. The null hypotheses of non-Granger causality from exchange rates to fundamentals are rejected in nine cases at the 5% significance level and five more cases at the 10% significance level. However, as illustrated in Table 3(b), the rejection frequency to the non-Granger causality null hypothesis is much less based on the bootstrap distribution than based on the asymptotic distribution. The null hypothesis that $\Delta f_t$ fails to cause $\Delta f_{t-1}$ is rejected in only two cases at the 5% significance level, and three more cases at the 10% significance level.

To summarize, the smaller the sample size is implemented in the test, the weaker the evidence of the Granger causality based on the bootstrap method is. For the full sample, the evidence of the Granger causality relationship between exchange rates and fundamentals from the bootstrap test is slightly weaker than the asymptotic test’s. However, the results of the bootstrap test with the later part of the sample are greatly different from the results of the asymptotic test. Little evidence for the Granger causality relationship between exchange rates and fundamentals is found in the bootstrap test. This suggests that the Granger causality relationship between exchange rates and fundamentals is not as significant as what [5] discovered. The existing positive evidence for the present-value model is greatly discounted.

**V. Size of the Tests**

In the previous section, we find that results of the Granger causality test from exchange rates to fundamentals based on two different test approaches are distinct when using small samples. Although the asymptotic test statistics constructed from the small sample data might suffer from the size distortion, it is not sufficient to draw the conclusion that the evidence from the bootstrap test is more convincing than the asymptotic test. In this section, we examine the robustness of the two test methods and examine whether the bootstrap test performs better than the asymptotic test in this particular application.

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**TABLE I. Bivariate Granger Causality Test Results—Full Sample: 1974:1–2001:3**

| A. Rejections at 1%(***), 5%(**), and 10%(*) Levels of $H_0$: $\Delta f_t$ Fails to Cause $\Delta f_{t-1}$ |
|---|---|---|---|---|---|
| Canada | France | Germany | Italy | Japan | United Kingdom |
| $1\Delta(m_t - m^*_t)$ | * | ** | *** | | |
| $2\Delta(p_t - p^*_t)$ | | | *** | *** | *** |
| $3\Delta(i_t - i^*_t)$ | | ** | *** | | |
| $4\Delta(m_t - m^*_t) - \Delta(y_t - y^*_t)$ | * | | | | |
| $5\Delta(y_t - y^*_t)$ | | | | | |

| B. Rejections at 1%(***), 5%(**), and 10%(*) Levels of $H_0$: $\Delta f_t$ Fails to Cause $\Delta f_{t-1}$ |
|---|---|---|---|---|---|
| Canada | France | Germany | Italy | Japan | United Kingdom |
| $1\Delta(m_t - m^*_t)$ | | | *** | ** | * |
| $2\Delta(p_t - p^*_t)$ | | ** | *** | ** | |
| $3\Delta(i_t - i^*_t)$ | | | *** | |
| $4\Delta(m_t - m^*_t) - \Delta(y_t - y^*_t)$ | | * | | | |
| $5\Delta(y_t - y^*_t)$ | | | | | |

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The size of the 10% test is tabulated in Fig. 1 to 3. The numeric number of 1 to 6 represents Canada, France, Germany, Italy, Japan, and the United Kingdom, respectively. The upper panel of the figures summarizes the size of the asymptotic tests, and the lower panel of figures summarizes the size of the bootstrap tests. Since the nominal size of the test is 10%, the ideal value of size of a test is 0.1. We see that, in the upper panel of Fig. 1, the size of the asymptotic test is slightly larger than 10%. For the early part of the sample, as the upper panel of Fig. 2 displays, size of the asymptotic test increases by a large percentage. For the later part of the sample, the upper panel of Fig. 3 shows that size of the asymptotic test raises in all fundamental measures and in all sample countries by a large percentage. Moreover, some magnitudes of the size of the asymptotic tests rise up to almost 40%.

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**TABLE II** Bivariate Granger Causality Test Results - Early Part of the Sample: 1974:1-1990:2

<table>
<thead>
<tr>
<th>(a) p-values for the Asymptotic Test Statistics</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Rejections at 1%(* * <em>), 5%(</em> <em>), and 10%(</em> ) Levels of $H_0$: $\Delta t_f$ Fails to Cause $\Delta f_I$</td>
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<tr>
<td>$\Delta(m_t - m_T)$</td>
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<td>$\Delta(p_t - p_T)$</td>
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<tr>
<td>$\Delta(i_t - i_T)$</td>
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<tr>
<td>$\Delta(m_t - m_T) - \Delta(y_t - y_T)$</td>
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<tr>
<td>$\Delta(y_t - y_T)$</td>
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(b) p-values for the Bootstrap Test Statistics

<table>
<thead>
<tr>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Rejections at 1%(* * <em>), 5%(</em> <em>), and 10%(</em> ) Levels of $H_0$: $\Delta t_f$ Fails to Cause $\Delta f_I$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta(m_t - m_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(p_t - p_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(i_t - i_T)$</td>
<td>***</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(m_t - m_T) - \Delta(y_t - y_T)$</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(y_t - y_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

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**TABLE III** Bivariate Granger Causality Test Results - Later Part of the Sample: 1990:3-2001:3

<table>
<thead>
<tr>
<th>(a) p-values for the Asymptotic Test Statistics</th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>United Kingdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Rejections at 1%(* * <em>), 5%(</em> <em>), and 10%(</em> ) Levels of $H_0$: $\Delta t_f$ Fails to Cause $\Delta f_I$</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta(m_t - m_T)$</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(p_t - p_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(i_t - i_T)$</td>
<td>***</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(m_t - m_T) - \Delta(y_t - y_T)$</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(y_t - y_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

(b) p-values for the Bootstrap Test Statistics

<table>
<thead>
<tr>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>United Kingdom</th>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta(m_t - m_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(p_t - p_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<td>*</td>
</tr>
<tr>
<td>$\Delta(i_t - i_T)$</td>
<td>***</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(m_t - m_T) - \Delta(y_t - y_T)$</td>
<td>**</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>$\Delta(y_t - y_T)$</td>
<td>*</td>
<td>*</td>
<td>*</td>
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</tr>
</tbody>
</table>
Figure 1. Kingdom, respectively. The upper panel and lower panel of the figures summarize the size of the asymptotic Granger causality tests and that of the bootstrap Granger causality tests from exchange rates to fundamentals, respectively.

In contrast, as what can be seen in the lower panels of Fig. 1 to Fig. 3, the size of the bootstrap test does not change much and remains stable at the nominal 10% significance level. Also, the size of the bootstrap test is lower than that of the asymptotic test in all samples. The Monte Carlo study shows that the asymptotic test has larger size distortion than...
the bootstrap test in this particular application. It implies that the asymptotic Granger causality test suffers the size distortion problem for the type of the data used in [5], and thus more direct evidence for the present-value model for exchange rates is required.

VI. CONCLUSION

This study is in response to the finding of the Granger causality relationship based on the asymptotic test in [5]. In [5], the authors use the present-value model to explain the finding of the close to random walk exchange rate in empirical studies. Although the empirical findings of the near-random walk exchange rate are consistent with the exchange rate behavior under their explanation, the findings are not sufficient to directly confirm the present-value model for exchange rates. The Granger causality test with the asymptotic method is implemented to validate the model. However, the type of data used in their study is very likely to lead to the size distortion in the test results because the sample size of the data is small.

This paper employs the bootstrap method to re-evaluate the evidence of the causality relationship between exchange rates and fundamentals. The bootstrap test results show that the evidence of Granger causality from exchange rates to fundamentals is not as significant as the existing evidence from the asymptotic method in all sample periods. Additionally, the Monte Carlo experiment results demonstrate that the bootstrap test performs better than the asymptotic test in respect of the robustness of the tests in this particular application. The large size in the asymptotic test shows that results in [5] are greatly distorted by the small-sample problem. Therefore, the existing Granger causality evidence is not strong enough to support the present-value model for exchange rate in [5].

More direct evidence is needed for the present-value model for exchange rates. One may explore long-horizon exchange rate predictability under the present-value model. For example, over the longer horizon, the near-one discount rate factor becomes smaller. In this case, one of the assumptions in [5] for the near random walk exchange rate fails, and it is likely to discover the exchange rate predictability over the long horizon.

REFERENCES


