A Hierarchical Fuzzy Linear Regression Model for Forecasting Agriculture Energy Demand: A Case Study of Iran

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Abstract. The aim of this paper is to develop a prediction model of energy demand of agriculture sector in Iran. A fuzzy-based approach is applied for the agriculture energy demand forecasting using socio-economic indicators. This approach is structured as a fuzzy linear regression (FLR). A hierarchical FLR model is designed properly. This paper indeed proposes a hierarchical FLR model by which the inputs to the ending level are obtained as outputs of the starting levels. Actual data from 1993-2006 is used to develop the hierarchical FLR and illustrate capability of the approach in this regard. The estimation fuzzy problem for the model is formulated as a linear optimization problem and is solved using the linear programming based simplex method. Furthermore, having obtained the fuzzy parameters, the agriculture energy demand is predicted from 2007 to 2020. The results provide scientific basis for the planned development of the energy supply of agriculture sector in Iran.

Keywords: Energy consumption, Agriculture sector, Forecasting, FLR

1. Introduction

According to the economical theories and views, energy is one of the main and the most important production factors in agriculture sector. Predicting its future consumption is an important step in macro-planning in agricultural and energy sections.

Conventional regression analyzes is one of the most used statistical tools to explain the variation of a dependent variable \( Y \) in terms of the variation of explanatory variables \( X \) as: \( Y = f(X) \) where \( f(X) \) is a linear function. The use of statistical conventional regression is bounded by some strict assumptions about the given data. This model can be applied only if the given data are distributed according to a statistical model and the relation between \( X \) and \( Y \) is crisp. Overcoming such limitations, fuzzy regression is introduced which is an extension of the conventional regression and is used in estimating the relationships among variables where the available data are very limited and imprecise and variables are interacting in an uncertain, qualitative and fuzzy way [1].

During the past decades, the applications of fuzzy regression models for energy forecasting problems have been resulted in several research papers [1-3]. In 2004 a fuzzy linear regression model had been developed by Al-Kandari et al for electric load forecasting. The estimation fuzzy problem for the model is turned out to a linear optimization problem, fuzzy linear regression. It has been found using such fuzzy model; a reliable operation for the electric power system could be obtained [2]. In 2009, Taghizadeh et al. have formulated a forecasting multi-level fuzzy linear regression model to predict the transport energy demand of Iran up to 2020 using socio-economic and transport related indicators [3]. Also in this year a
flexible fuzzy regression model have been formulated by Azadeh et al. to forecast oil consumption based on standard economic indicators [1].

In this paper agriculture energy demand of Iran is forecasted using FLR model considering economic and social indicators for the time span 2008 to 2020. For the estimation, time series data covering the period 1993 to 2007 are used. The remaining parts of the paper are organized as follows. In section 2, FLR model is introduced. Details of the proposed forecast strategy and numerical results are described in section 3. A brief review of the paper is given in section 4.

2. Fuzzy linear regression model

Fuzzy linear regression model was proposed by Tanaka et al [4]. This method is widely applied to various applications including marketing, management and sales forecasting. It can also be applied for energy forecasting problems. In this section, a formulation for fuzzy linear regression estimation problem is presented. In this model, the outputs are non-fuzzy observations. Also, the inputs are non-fuzzy inputs. The base model is assumed to be a fuzzy linear function as below:

\[ y = f(x, \tilde{A}) = \tilde{A}_0 + \tilde{A}_1 X_1 + \tilde{A}_2 X_2 + \ldots + \tilde{A}_n X_n \]  

where \( \tilde{A}_i (i = 1, 2, \ldots, n) \) are the fuzzy coefficients in the form of \((p_i, c_i)\) where \( p_i \) is the middle and \( c_i \) is the spread. The membership functions for each type of \( \tilde{A}_i \) are assumed a triangular membership. So it can be expressed by definition 1 as:

\[ \tilde{A}_i(a) = \begin{cases} \frac{a - p_i}{c_i}, & p_i - c_i \leq a \leq p_i + c_i \\ 0, & \text{otherwise} \end{cases} \]  

By applying the Extension Principle [5] the membership function of fuzzy number \( \tilde{y} \) is given by:

\[ \tilde{y}(y) = \begin{cases} \max(\min\{\tilde{A}_i(a_i)\}) & \{a_i, y = f(x, a_i)\} \neq \emptyset \\ 0, & \text{otherwise} \end{cases} \]  

From (1) and (3) we get:

\[ \tilde{y}(y) = \begin{cases} \frac{y - (p_0 + \sum_{i=1}^n p_i x_i)}{1 - \sum_{i=1}^n c_i |x_i|} & x_j \neq 0 \\ \frac{c_0 + \sum_{i=1}^n c_i |x_i|}{1 - \sum_{i=1}^n c_i |x_i|} & x_j = 0, y = 0 \\ 0 & x_j = 0, y \neq 0 \end{cases} \]  

The spread of \( \tilde{y} \) is \( \sum c_i |x_i| \) and the middle of \( \tilde{y} \) is \( \sum p_i |x_i| \).

We seek to find the coefficients \( \tilde{A}_i = (p_i, c_i) \) that minimize the spread of the fuzzy output for all data sets. Eq. (5) shows the objective function [6].

\[ \text{Min} \sum_{j=1}^m \sum_{i=1}^n (c_0 + \sum_{i=1}^n c_i |x_{ij}|) \]  

and the constraints require that each observation \( y_j \) has at least h degree of belonging to \( \tilde{y}(y) \), that is:[7]

\[ \tilde{y}(y_j) \geq h \quad j = 1, 2, \ldots, m \]  

The degree \( h \) is specified by the user.

By substituting Eq. (4) into Eq. (6), it is obtained:

\[ y_j \geq p_0 + \sum_{i=1}^n \frac{p_i x_{ij} - (1-h)(c_0 + \sum_{i=1}^n c_i |x_{ij}|)}{1 - \sum_{i=1}^n c_i |x_i|}, \quad j = 1, 2, \ldots, m \]  

\[ y_j \leq p_0 + \sum_{i=1}^n \frac{p_i x_{ij} + (1-h)(c_0 + \sum_{i=1}^n c_i |x_{ij}|)}{1 - \sum_{i=1}^n c_i |x_i|}, \quad j = 1, 2, \ldots, m \]  

The above analyzes leads to the following linear programming problem [2]:

\[ \text{Min} \sum_{j=1}^m \sum_{i=1}^n (c_0 + \sum_{i=1}^n c_i |x_{ij}|) \]  

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\[ \text{Min} \sum_{j=1}^m \sum_{i=1}^n (c_0 + \sum_{i=1}^n c_i |x_{ij}|) \]  

and the constraints require that each observation \( y_j \) has at least h degree of belonging to \( \tilde{y}(y) \), that is:[7]

\[ \tilde{y}(y_j) \geq h \quad j = 1, 2, \ldots, m \]  

The degree \( h \) is specified by the user.
\[
\text{Min } \sum_{j=1}^{m} \left( c_0 + \sum_{i=1}^{n} c_i |x_j| \right)
\]

s.t.
\[
y_j \geq p_0 + \sum_{i=1}^{n} p_i x_{ij} - (1 - h)(c_0 + \sum_{i=1}^{n} c_i |x_j|) \quad j = 1, 2, \ldots, m
\]
\[
y_j \leq p_0 + \sum_{i=1}^{n} p_i x_{ij} + (1 - h)(c_0 + \sum_{i=1}^{n} c_i |x_j|) \quad j = 1, 2, \ldots, m
\]
\[
c_i \geq 0, \quad p_i \geq 0
\]

3. The hierarchical FLR model development and application

In this section agriculture energy demand of Iran from 2008 to 2020 is forecasted regarding socio-economic indicators using a hierarchical fuzzy linear regression model. The structure of the designed hierarchical FLR is given in Fig. 1.

The main FLR (FLR5) takes agriculture energy demand in the last year, value added of agriculture sector, growth rate of population, growth rate of gas oil price and growth rate of electricity price as inputs and produces the agriculture energy demand. The inputs to ending level are obtained as outputs of the starting levels. The value added of agriculture sector, population, gas oil price and electricity price are forecasted using FLRs. Table 1 summarizes the FLRs inputs and output.

<table>
<thead>
<tr>
<th>FLR</th>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>value added of agriculture sector in the last year, value added of agriculture sector in two last year</td>
<td>value added of agriculture sector</td>
</tr>
<tr>
<td>2</td>
<td>population in the last year, population in two last year,</td>
<td>Population</td>
</tr>
<tr>
<td>3</td>
<td>Gas oil price in the last year, gas oil price in two last year</td>
<td>gas oil price</td>
</tr>
<tr>
<td>4</td>
<td>electricity price in the last year, electricity price in two last year</td>
<td>electricity price</td>
</tr>
<tr>
<td>5</td>
<td>value added of agriculture sector, growth rate of population, growth rate of gas oil price, growth rate of electricity price, agriculture energy demand in the last year</td>
<td>agriculture energy demand</td>
</tr>
</tbody>
</table>

By this way fuzzy linear regression models are developed for forecasting of value added of agriculture sector, population, gas oil price, electricity price and agriculture energy demand. The FLR models are presented in Table 2.

where \( AGVA \) is value added of agriculture sector, \( POP \) is population, \( AGPGO \) is gas oil price, \( AGPEL \) is electricity price, \( AGENG \) is agriculture energy demand, \( \alpha \) is growth rate.
Data related with agriculture energy demand model is collected from Institute for International Energy Studies (IIES) and Iran Ministry of Energy. All values given for the economic variables are normalized based on the fixed prices of 1997 (1997=100).

**TABLE II. FLR MODELS FOR AGRICULTURE ENERGY DEMAND FORECASTING.**

<table>
<thead>
<tr>
<th>FLR</th>
<th>FLR models</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$AGVA(t) = (p_0, c_0) + (p_1, c_1)AGVA(t-1) + (p_2, c_2)AGVA(t-2)$</td>
</tr>
<tr>
<td>2</td>
<td>$POP(t) = (p_0, c_0) + (p_1, c_1)POP(t-1) + (p_2, c_2)POP(t-2)$</td>
</tr>
<tr>
<td>3</td>
<td>$AGPGO(t) = (p_0, c_0) + (p_1, c_1)AGPGO(t-1) + (p_2, c_2)AGPGO(t-2)$</td>
</tr>
<tr>
<td>4</td>
<td>$AGPEL(t) = (p_0, c_0) + (p_1, c_1)AGPEL(t-1) + (p_2, c_2)AGPEL(t-2)$</td>
</tr>
<tr>
<td>5</td>
<td>$AGENG(t) = (p_0, c_0) + (p_1, c_1)AGVA(t) + (p_2, c_2)POP(t) + (p_1, c_1)\alpha AGPGO(t) + (p_2, c_2)AGPEL(t) + (p_1, c_1)\alpha AGENG(t-1)$</td>
</tr>
</tbody>
</table>

By using the past history data fuzzy linear regression equation of agriculture energy demand of Iran is as bellow:

$$AGENG(t) = (7/572,0) + (0/0003,0)AGVA(t) + (0,1/032)\alpha POP(t) + (0/0618,0/010)\alpha AGPGO(t) + (0/002,0)\alpha AGPEL(t) + (0/194,0)AGENG(t-1)$$

Average absolute error percentage (AAEP) criteria is used for validity investigation of the model. The AAEP is calculated from the following equation:

$$AAEP = \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{x}(i) - x(i)}{x(i)}$$

where $\hat{x}(i)$ is the estimated data and $x(i)$ is the actual data. The AAEP value is 2.98%. That is acceptable.

For forecasting agriculture energy demand in future, value added of agriculture sector, population, gas oil price and electricity price are forecasted. The fuzzy linear regression is carried out to find fuzzy parameters. The results are shown in Table 3.

**TABLE III. FUZZY PARAMETERS OF FLR1,2,3,4 MODELS.**

<table>
<thead>
<tr>
<th>FLR</th>
<th>Fuzzy parameters</th>
<th>i = 0</th>
<th>i = 1</th>
<th>i = 2</th>
<th>FLR</th>
<th>Fuzzy parameters</th>
<th>i = 0</th>
<th>i = 1</th>
<th>i = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_l$</td>
<td>0</td>
<td>0.662</td>
<td>0.374</td>
<td>3</td>
<td>$p_l$</td>
<td>33.490</td>
<td>0.486</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$c_l$</td>
<td>0</td>
<td>0.092</td>
<td>0</td>
<td>3</td>
<td>$c_l$</td>
<td>15.413</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$p_l$</td>
<td>0</td>
<td>0.226</td>
<td>0.796</td>
<td>4</td>
<td>$p_l$</td>
<td>0</td>
<td>0.662</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td>$c_l$</td>
<td>640.999</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>$c_l$</td>
<td>0</td>
<td>0.092</td>
<td>0</td>
</tr>
</tbody>
</table>

The AAEP values of value added of agriculture sector, growth rate of population, growth rate of gas oil price and growth rate of electricity price related to fuzzy models can be seen in Table 4. The AAEP values are acceptable.

**TABLE IV. AAEP VALUES OF FLR1,2,3,4 MODELS.**

<table>
<thead>
<tr>
<th>FLR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4/13%</td>
<td>0.68%</td>
<td>13.46%</td>
<td>14.44%</td>
</tr>
</tbody>
</table>

Then the fuzzy models are used to predict value added of agriculture sector, population, gas oil price and electricity price from 2007 to 2020. The estimation of value added of agriculture sector, population, gas oil price and electricity price are given in Fig. 2, 3, 4 and 5. These graphs show the actual data versus the FLR results. The value added of agriculture sector will reach to a level of $91746 \times 10^9$ R, population is about 85 million, gas oil price is about 65 R/L and electricity price is about 6 R/KWH in middle level in 2020.
By using equation (9) and related data the agriculture energy demand is estimated. The results can be seen in Table 5. Agriculture energy demand will reach to a level of 47.87 MBOE in 2020.

<table>
<thead>
<tr>
<th>Years</th>
<th>agriculture energy demand (MBOE)</th>
<th>Years</th>
<th>agriculture energy demand (MBOE)</th>
<th>Years</th>
<th>agriculture energy demand (MBOE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>36.37</td>
<td>2012</td>
<td>40.67</td>
<td>2017</td>
<td>44.98</td>
</tr>
<tr>
<td>2008</td>
<td>37.97</td>
<td>2013</td>
<td>41.48</td>
<td>2018</td>
<td>45.92</td>
</tr>
<tr>
<td>2009</td>
<td>38.58</td>
<td>2014</td>
<td>42.31</td>
<td>2019</td>
<td>46.88</td>
</tr>
<tr>
<td>2010</td>
<td>39.21</td>
<td>2015</td>
<td>43.18</td>
<td>2020</td>
<td>47.87</td>
</tr>
<tr>
<td>2011</td>
<td>39.91</td>
<td>2016</td>
<td>44.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusions

This paper focused on forecasting the annual agriculture energy demand of Iran regarding socio-economic indicators using hierarchical fuzzy linear regression model. A hierarchical fuzzy linear regression model was designed to take agriculture energy demand in the last year, value added of agriculture sector, growth rate of population, growth rate of gas oil price and growth rate of electricity price as inputs and produces agriculture energy demand. The value added of agriculture sector, growth rate of population, growth rate of gas oil price and growth rate of electricity price were forecasted using FLR models. Actual data from 1993 to 2007 were used and agriculture energy demand of Iran from 2007 to 2020 was forecasted.

5. Acknowledgements

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6. References


